**Virtual Simulation Equations**

**1.input Parameters**

Physical Constants

Gravitational acceleration (g) 9.812 m/sec^2

Mathematical constant (π) 3.14159

Air density (ρ) 1.225 m/sec^3

Time step (ΔT) 0.5 sec [or whatever you set]

Zero speed tolerance (εv) 0.01 m/sec

Vehicle Parameters

Gross vehicle mass (m) 159 kg

Rear tire diameter (D) 0.4064 m

Tire/road friction coefficient (μ) 0.75

Tire rolling resistance coefficient (Crr) 0.017

Wheel efficiency (ηw) [1 for solid wheel, 1 to 0.9in 1 hr for spoked]

Drag coefficient (Cd) 0.45

Frontal area (A) 1.6 m^2

Battery Parameters

R0+R2 0.02 Ohm

R1 0.010546 Ohm

τ 3000 sec

ΔT/τ 1.667E-04

Maximum battery capacity (C) 26 Ah

Throttle Parameters

Maximum throttle value (THmax) 5

Percent of torque at maximum regen (THregn) 150 %

Max motor RPM for regen scaling (RPMmax) 750

Sprocket/Chain Parameters

Wheel sprocket teeth count (Tsp) 8

Motor sprocket teeth count (Tm) 15

Gear ratio (N) [1 for wheel motor, Tm/Tsp for sprocket/chain]

Gear efficiency (ηg) [1 for wheel motor, 0.9 for sprocket/chain]

**2. Simulation Parameters**

Time (t) 0, ΔT, 2ΔT, 3ΔT, …..

Throttle value (TH) -1, 0, 1, 2, 3, 4, 5

Maximum Torque (TRMAX) See equation 1

Maximum Current (IMAX) See equation 2

Motor Torque (TRMOTOR) See equation 3

Motor Current (IMOTOR) See equation 4

Tire Patch Force (FTIRE) See equation 5

Rolling Resistance Force (FRR) See equation 6

Drag Force (FD) See equation 7

Net Tire Patch Force (FNET) See equation 8

Acceleration (ACCEL) See equation 9

Speed (SPD) See equation 10

Position (POS) See equation 11

Motor RPM (RPM) See equation 12

Currentin R1 (IR1) See equation 13

Voltagein R0/R2 (VR0R2) See equation 14

Voltagein R1 (VR1) See equation 15

Open Circuit Voltage (VOC) See equation 16

Battery Voltage (VBATT) See equation 17

Zero Load Voltage (VZEROL) See equation 18

Actual State of Charge (SOC) See equation 19

Zero Load SOC (SOCZEROL) See equation 20

Energy Draw (E) See equation 21

RPM/V scaling (RPMV) See equation 22

Battery Output Power (PBATT) See equation 23

Motor Output Power (PMOTOR) See equation 24

Vehicle Power (PVEH) See equation 25

**3. Cycle Analyst Display**

Speed in KPH (SKPH) See equation 26

Speed in MPH (SMPH) See equation 27

Energy Draw (ECC) See equation 28 [depends on E above and if they reset last reading]

**4. Initial Parameter Settings**

All parameters set to zero except voltages and SOC as follows:

VOC = VBATT = VZEROL = 12.6631

SOC = SOCZEROL = 100%

**5. Simulation Steps**

In the simulation the previous step is *i*, and the new step being calculated is *i+1*. So in 4. above we are setting t*i* = 0, TH*i* = 0, TRMAX*i* = 0, etc.

Calculations are done in this sequence:

t*i+1* = t*i* + ΔT

TH*i+1* is given based on the throttle setting at the location on the track

TRMAX*i+1* = -0.000085488\*RPMV*i*^6+0.00390688\*RPMV*i*^5-0.0736331\*RPMV*i*^4+0.710122\*RPMV*i*^3-3.19602\*RPMV*i*^2-1.24086\*RPMV*i*+81.6265 Eq 1

IMAX*i+1* = -0.6174\* RPMV*i* + 41.469 IF RPMV*i* ≤ 13.79361, otherwise

IMAX*i+1* = -17.99\* RPMV*i* + 281.35 Eq 2

TRMOTOR*i+1* = TRMAX*i+1* \* TH*i+1* \* THregn \* (RPM*i* / RPMmax)^2 IF TH*i+1* < 0, otherwise

TRMOTOR*i+1* = 0 IF SOC*i* = 0, otherwise

TRMOTOR*i+1* = TRMAX*i+1* \* (TH*i+1* / THmax)^1.6 Eq 3

IMOTOR*i+1* = IMAX*i+1* \* TH*i+1* \* THregn \* (RPM*i* / RPMmax)^2 IF TH*i+1* < 0, otherwise

IMOTOR*i+1* = 0 IF SOC*i* = 0, otherwise

IMOTOR*i+1* = IMAX*i+1* \* (TH*i+1* / THmax)^1.6 Eq 4

FTIRE*i+1* = TRMOTOR*i+1* / (D/2) \* ηg Eq 5

FRR*i+1* = m \* g \* Crr / ηw Eq 6

FD*i+1* = 0.5\*ρ\*Cd\*A\*SPD*i*^2 Eq 7

FNET*i+1* = 0 IF SPDi = 0 AND FTIRE*i+1* FRR*i+1,* otherwise

FNET*i+1* = FTIRE*i+1 -* FRR*i+1 -* FD*i+1* Eq 8

ACCEL*i+1* = FNET*i+1* / m Eq 9

SPD*i+1* = ACCEL*i+1* \* ΔT + SPD*i*

IF SPD*i+1* εv*,* THEN SPD*i+1* = 0 Eq 10

POS*i+1* = SPD*i+1* \* ΔT + POS*i* Eq 11

RPM*i+1* = SPD*i+1* \* 60 / (D\* π) Eq 12

IR1*i+1* = ΔT/τ \*IMOTOR*i+1* + (1 - ΔT/τ) \* IR1*i* Eq 13

VR0R2*i+1* = IMOTOR*i+1* \* (R0+R2) Eq 14

VR1*i+1* = IR1*i+1* \* R1 Eq 15

VOC*i+1* = 0.0000030802\* SOCZEROL*i* ^3 - 0.00068882\* SOCZEROL*i* ^2 + 0.056091\* SOCZEROL*i* + 10.862 Eq 16

VBATT*i+1* = VOC*i+1* - VR0R2*i+1* – VR1*i+1* Eq 17

VZEROL*i+1* = VOC*i+1* - VR1*i+1* Eq 18

SOC*i+1* = 28.752 \* VZEROL*i+1*^3 - 973.093 \* VZEROL*i+1*^2 + 10988.9 \* VZEROL*i+1* - 41397.3226448

IF SOC*i+1* > 100 THEN SOC*i+1* = 100

IF SOC*i+1* < 1 THEN SOC*i+1* = 0 Eq 19

SOCZEROL*i+1* = (1 – E*i* / C)\*100

IF SOCZEROL*i+1* > 100 THEN SOCZEROL*i+1* = 100 Eq 20

E*i+1* = (IMOTOR*i+1* \* ΔT / 3600) + E*i* Eq 21

RPMV*i+1* = RPM*i+1* / (VBATT*i+1* \* 4) Eq 22

PBATT*i+1* = IMOTOR*i+1* \* VBATT*i+1* \* 4 Eq 23

PMOTOR*i+1* = TRMOTOR*i+1* \* RPM*i+1* \*2 \* π / 60 Eq 24

PVEH*i+1* = (FRR*i+1* + FD*i+1*) \* SPD*i+1* Eq 25

SKPH*i+1* = SPD*i+1* \*3600 / 1000 Eq 26

SMPH*i+1* = SKPH*i+1*/ 1.6 Eq 27

ECC*i+1* = ECC*i* + (E*i+1* - E*i*) Eq 28

**5. Battery Charging**

If the team hits the battery charge button when they are allowed (i.e. any time prior to the start of the race) then set

VOC*i* = VBATT*i* = VZEROL*i* = 12.6631

SOC*i* = SOCZEROL*i* = 100%

E*i* = 0

**NOTE: Vehicle speed must be 0 when they hit this button otherwise nothing happens**

**6. Braking on turns**

The algorithm for checking if the vehicle is going too fast on a turn and applying regen (+ additional mechanical braking) is as follows:

Step 1: calculate instantaneous turning radius R*i*. Take the raceline coordinates (X*i+1*,Y*i+1*), (X*i*,Y*i*), (X*i-1*,Y*i-1*) and plug them into an algorithm that determines the radius R of a circle when three points on the circumference are defined. Note that this is done after Eq 11 has been solved and we have POS*i+1* so we can obtain the coordinates (X*i+1*,Y*i+1*) and we can check the speed in step 2. We refer to this as R*i* and not R*i+1* because the radius is more accurately representative of the center coordinate of the three points (X*i+1*,Y*i+1*), (X*i*,Y*i*), (X*i-1*,Y*i-1*). Of course we don’t expect R to change a lot if we shift by +/- one point. Assuming ΔT = 0.1 sec in the actual race simulation then at max speed the vehicle moves 1.2 m each step.

Step 2: calculate the maximum allowable speed SPDMAX*i+1* = SQRT(g\*μ\*R*i*)

Step 3: If SPD*i+1* ≤ SPDMAX*i+1* then proceed to Eq 12. If not then:

TH*i+1* = -1

Solve Equations 1 to 4

Equation 5 becomes: FTIRE*i+1* = FRR*i* + FD*i* + m\*((SPDMAX*i+1* – SPD*i*) / ΔT) Eq 5

Step 4: Continue with Eq 6 and beyond normally